RESEARCH DEPARTMENT

A method for estimating the protection required by a television service against interference from several sources

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THE BRITISH BROADCASTING CORPORATION
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RESEARCH DEPARTMENT

A METHOD FOR ESTIMATING THE PROTECTION REQUIRED BY A TELEVISION SERVICE AGAINST INTERFERENCE FROM SEVERAL SOURCES

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April 1963

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SUMMARY

An engineer planning a television service in a channel already occupied by other transmissions must be able to estimate the combined effect of interference from several sources. Severity of interference is judged subjectively, and an ideal method of making the estimate would be one which gave the answer directly in terms of a subjective judgment. This is, however, not possible. Computations must rely on objective quantities, such as voltage, or power, or probability, and the best that can be expected of such a procedure is that the severity of the interference estimated objectively will match closely the severity which a subjective judgment would attribute to it.

The method described here works in probabilities, because it is believed that this quantity is the most closely related to a subjective judgment on the severity of interference. Variations in signal strength occur in time as well as with location, and other methods of making probability estimates which have come to the notice of the author assume a correlation coefficient of unity or zero for one or other of these variations. These assumptions are not valid, the correlation being partial in both cases. The system evolved has therefore been placed in a time-location frame of reference to take cognisance of this fact.

1. GENERAL

There are two very different aspects of a broadcast transmission, depending on whether the location at which it is received lies inside or outside the service area. Within the area, it is a source of information and entertainment, but outside it, at locations tuned to its frequency, it is a source of interference. The number of broadcast transmissions of all sorts is continually increasing though the spectrum available to each is limited. A situation is therefore developing, particularly in the case of television, in which the number of broadcasts sharing a common channel becomes progressively greater, and mutual interference between them more probable.

Considerations of mutual interference between television transmissions raise problems that are special to this form of broadcasting. The presence of line and field frequencies in addition to the carrier frequency, and the line-by-line display of the picture, all contribute to the subjective effect which an interfering signal may produce. These effects are measured by the ratio by which the strength of the wanted radio-frequency signal must exceed that of an unwanted radio-frequency signal in order that the viewers' enjoyment of the picture may not be marred by

interference. A number of laboratory experiments have been carried out by various bodies to measure this ratio, and desirable levels of protection[1] have been internationally agreed. There are also internationally accepted propagation curves[2] which give, for the different frequency bands and for various probabilities with respect to time, the field strengths which may be exceeded at given distances from the source of the interference. By means of these two data, the wanted field strength protected against interference from a single unwanted transmission for a specified percentage of the time may be estimated.

Methods have been proposed, mainly in F.C.C. documents,[3] for estimating the combined effect of interference from many sources, but the estimates of interference that have hitherto been made in practice have assumed that only the preponderant unwanted signal need be considered. This assumption loses its validity when two or more interferences of comparable field strength are present, a situation which will become increasingly frequent with the expansion of television. It is to meet the need of such situations that the C.C.I.R. recently adopted[1] for use at the European Broadcasting Conference, Stockholm, 1961, a method, based on one of the F.C.C. proposals, of estimating the combined effect of many sources of interference. In this method, which has become known as the Simple Multiplication (or S.M.) method, the overall (location) probability of the protection given to a certain field strength for a chosen percentage of the time is calculated as the product of the separate location probabilities that it will be protected against interference from each source.

2. ASSUMPTIONS IN THE S.M. METHOD

It is usual in propagation studies to assume that variations of field strength with location and with time are each log-normal in distribution. This assumption is basic to the S.M. method, as it is to all methods involving estimates of the probability of interference. All such methods depend on the principle that the probability of several uncorrelated events occurring simultaneously is the product of their individual probabilities. Where the methods differ is in their treatment of the correlation between the wanted and unwanted signals. The S.M. method assumes that the variations are uncorrelated with respect to location and correlated with respect to time. Field trials carried out by G.F. Swann of the Post Office Engineering Department and described in the next paragraph do not, however, give support to this view.

3. CORRELATION BETWEEN TWO SIGNALS VARYING WITH LOCATION

Field strength measurements were made in a large number of locations in typical residential areas in types of terrain ranging from flat to moderately undulating. A dipole aerial was used to receive two vertically polarized transmissions, so that the two signals under investigation could be measured using the same aerial position at each location. Analysis of the several hundred pairs of values obtained gave correlation coefficients varying from 0·38 for signals arriving from opposite directions to 0·78 for signals arriving from the same direction.

While these particular tests were confined to television transmissions in Band III, experience in other bands supports the conclusion that, for purposes of calculation, a correlation coefficient of 0·5 is more representative of average conditions throughout a television service area than the zero correlation which is often assumed to exist.

4. CORRELATION IN TIME

It is well known that even at the low end of the v.h.f. band, no correlation exists between the short-term fading observed on two receivers when the receiving aerials are one or two hundred metres apart. Reciprocally, therefore, there is no correlation between the short-term variations of separate interfering signals at a single site when the sources are remote from each other. Over longer periods of time, however, of durations varying from hours to days, correlation exists, since in conditions of abnormal propagation, the probability of interference from all sources increases approximately in the same degree. Thus the overall coefficient of correlation between the time variations of two signals lies somewhere between 0 and 1.

A short analysis was carried out to get some idea of the degree of correlation that might exist in certain conditions. A comparison was made of 96 hourly median values taken from field strength records of two Band II signals received at a single site. The paths were 77 and 100 miles long (124 and 161 km respectively), and the transmitters were separated by 146 miles (235 km). The coefficient of correlation obtained was 0·49, which for 96 pairs of values is a significant result. The two records could not be synchronized reliably to compare instantaneous values, but had this been possible, the value of the coefficient would certainly have been near zero. On the other hand, had the medians of a more extended period been compared, such as a day, or even a week, a coefficient approaching unity would have resulted. Thus, time correlation depends on the period over which the comparison is made. For television interference, the period of one hour, and consequently the value of 0·5, seems a reasonable one to take.

5. THE RELATIVE STANDARD DEVIATION FOR CORRELATED EVENTS

The principle that the probabilities of uncorrelated events may be multiplied together can be extended to partially correlated events if an appropriate value of standard deviation is used. The standard deviation σ_0 of the ~~sum~~ difference of two varying quantities, when some correlation exists between them, is given by the expression

$$\sigma_0 = [\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2]^{1/2}$$

where σ_1 and σ_2 are respectively the standard deviations of the two variables and ρ is the coefficient representing the degree of correlation between them. The representative value for ρ for variations of signal strength with respect both to location and to time has been shown to be 0·5. If $\sigma_1 = \sigma_2$, a reasonable assumption, it is easy to see that

$$\sigma_0 = \sigma_1 = \sigma_2$$

Thus in problems of multiple interference, the preferred value for the 'modified' standard deviation is the measured standard deviation of the interference.

6. THE TIME-LOCATION FRAME OF REFERENCE

If field strength measurements are made by scanning an area over an extended period of time, the values obtained would be distributed in time-location. It is, however, convenient to measure field strength variations separately with respect to

location and to time, and this procedure results in the collected data falling into two groups, related respectively to the location and to the time frames of reference. In a service area, over an extended period of time, both aspects of the signal variations are always present, and it is therefore logical that the estimate of the protection provided should be made with respect to the variations of signal strength in time-location.

In discussing the various possible ways of estimating the combined effect of many interferences, the F.C.C. mentions[3] the time-location frame of reference, but suggests that it is unsuitable, because the individual location and time probabilities are not precisely separable in the answer obtained. Not only does the author not consider this a disadvantage, but he hopes that when the characteristics of protection in time-location have been described, it will be clear that an estimate arrived at in this way provides a better appreciation of the sort of service being considered than does a rigid adherence to fixed probabilities of protection in time and with location.

7. STANDARD DEVIATION OF SIGNAL VARIATIONS IN TIME-LOCATION

Signal variations with respect to time and to location are, for all practical purposes, uncorrelated. The standard deviation of the distribution of field strength over an infinite area and an infinite period of time is therefore the root-sum-square of the separate standard deviations. This is interpreted for the present purpose to be the standard deviation of the time-location variations of the unwanted field strength within the service area, its value being calculated from the separate location and time standard deviations obtained by experiment. If the standard deviation with respect to location is 8 dB, and if it is also 8 dB with respect to time (values which will later be seen to relate to tropospheric field strengths in Bands I and III), the standard deviation in time-location is $(8^2 + 8^2)^{1/2} = 11{\cdot}3$ dB, and this is the value used in calculating multiple interference over land paths in Bands I and III.

8. RELATIONSHIP BETWEEN STANDARDS OF PROTECTION IN DIFFERENT FRAMES OF REFERENCE

Let $E_{50,\ 50}$ (dB) be a median field strength with respect both to location and to time. Let the deviation x dB, within the time distribution of field strength, represent a probability t, and a deviation y dB within the location distribution represent a probability s; then

$$E_{s,\ t}\,(\text{dB}) = \{E_{50,\ 50}\,+\,(x + y)\}\ (\text{dB})$$

The time-location probability is the probability represented by a deviation $(x + y)$ in a distribution with a standard deviation which is the root-sum-square of the separate location and time standard deviations. Let this probability be u, so that

$$E_{s,\ t}\ (\text{dB}) = E_u\ (\text{dB})$$

It is clear from the fact that for any value of $(x + y)$ there is an infinite number of pairs of values x, y, that there is also an infinite number of pairs of probabilities s and t in location and in time respectively which are equivalent to the time-location probability u.

The relationship between the probabilities *s*, *t* and *u* is plotted in Fig. 1 for standard deviations of 8 dB for location variations and time variations, and 11·3 dB for variations in time-location. The upper scale gives the percentage probability in time-location. Probability in time alone is given by the ordinate scale, and location probability by the family of curves. It is seen, for instance, that protection at 50% of locations for 99% of the time is equivalent to a little over 95% protection in time-location. It is also equivalent to protection at 60% of locations for 98·1% of the time, or to protection for any other pair of location and time probabilities given by the ordinate through 95% on the time-location probability scale. Thus by specifying probability in time-location, it is possible to examine the ranges over which the separate location and time probabilities vary when the protection ratio is kept constant. It is clearly an advantage, when considering the degree of immunity from interference possessed by a service designed to protect 50% of locations for 99% of the time, to know that 80% of locations are being protected for no less than 93% of the time.

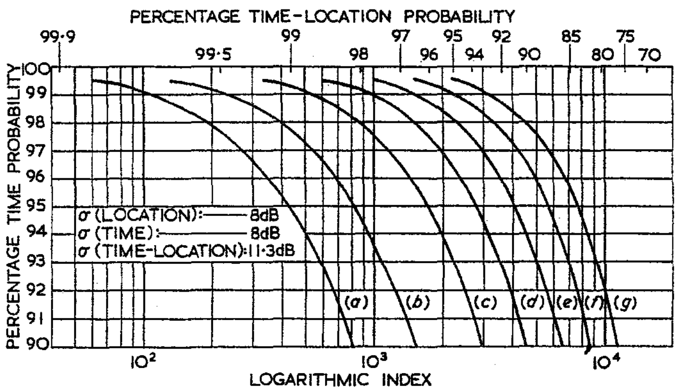

Fig. 1
Time-location probability and logarithmic index for various location and time probabilities

(*a*) 95%
(*b*) 90%
(*c*) 80%
(*d*) 70%
(*e*) 60%
(*f*) 50%
(*g*) 40%

Percentage location probability

9. LOGARITHMIC INDEX

The product of several probabilities is most easily calculated by summing their logarithms. When five-figure tables are used, the procedure is further simplified if the logarithms are multiplied by a factor of -10^5. The resulting number, which is positive and whole, is given the name 'Logarithmic Index'. For instance, one may derive by inspection from a table of five-figure logarithms that the logarithmic index for a probability 0·99 is 436 or for 0·95 is 2228. A further simplification is achieved by using a table of co-logarithms, from which a direct reading of the index may be obtained.

Table 1 is a table of probabilities for a standard deviation of 11·3 in steps of 0·5 dB. The figures in the first column are the deviations from the median value of field strength, the percentage probability of each deviation which is just not exceeded appearing in the middle column. To simplify the operation of taking the product, the logarithmic index for each probability is given in the third column. For the product, the appropriate logarithmic indices are added together, the result being the logarithmic index of the standard of protection achieved. The equivalent probability in time-location may be found from Table 1 or Fig. 1, and the equivalent pairs of probabilities in location and time from Fig. 1. The greater the probability of protection, the smaller will be the value of the index.

10. THE C.C.I.R. TROPOSPHERIC WAVE PROPAGATION CURVES[2]

The C.C.I.R. overland tropospheric wave propagation curves for Bands I, II and III, for a transmitting aerial height of 300 m and an effective radiated power (e.r.p.) of 1 kW, are reproduced in Fig. 2. The distribution of field strength with location in these bands is given in Fig. 3. It is seen from the latter that the standard deviation of the location variations is 8 dB. The standard deviation of the variations with time may be derived from the curves of Fig. 2 and is dependent on the separations between them. These are, for all practical purposes, constant throughout their length, so that the standard deviation is single-valued and independent of distance. It is well represented by 8 dB.

A further set of C.C.I.R. curves for oversea propagation is reproduced in Fig. 4, but these will be discussed in Section 12.

11. A PRACTICAL EXAMPLE

Table 2 illustrates by an example in Band I both the calculating procedure and the form in which this may most easily be carried out. The first seven columns contain data relating to the unwanted transmissions. In column 8 the median tropospheric field strength in dB(μV/m) for an e.r.p. of 1 kW is entered. The differences between the tabulated values and those that may be read from the 50% curve of Fig. 2 are due to the fact that corrections for the transmitting aerial height are included in the tabulated figures. The required protection ratio is given in column 9. The algebraic sum of the figures in columns 7, 8 and 9 is given in column 10; it is the field strength protected against median (location and time) interferences. Clearly this standard of protection is too low, and columns 11, 12 and 13 are introduced to enable the probable protection at different levels of field strength to be investigated. In this example the field strengths taken are 64, 65 and 66 dB(μV/m).

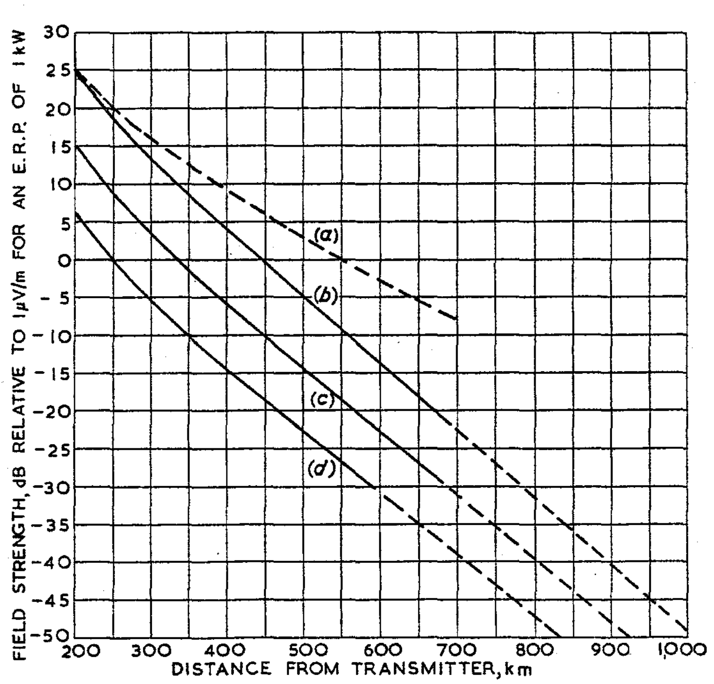

Fig. 2
Field strength of the tropospheric wave in Bands I, II and III

(*a*) for 1% of the time (oversea propagation)

(*b*) for 1% of the time (overland propagation)

(*c*) for 10% of the time (overland propagation)

(*d*) for 50% of the time (overland propagation)

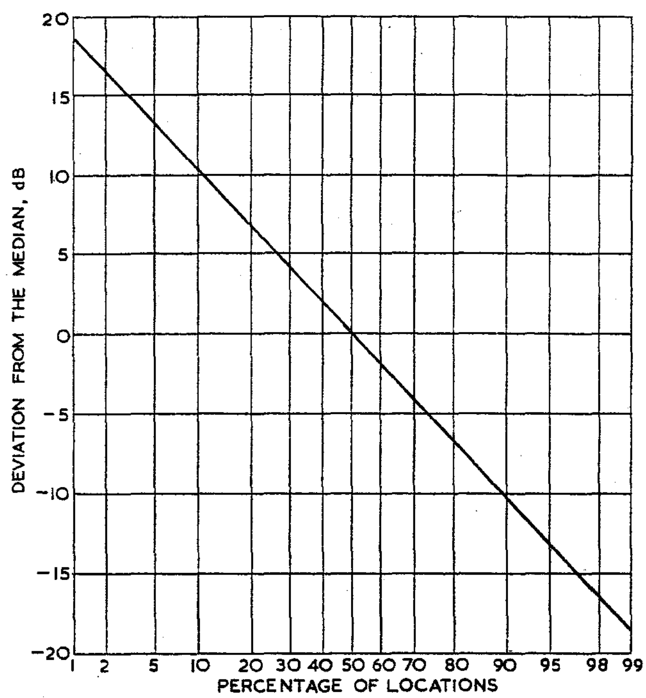

Fig. 3
Distribution of field strength with location, relative to the median value. Bands I, II and III

12. CURVES WITH STANDARD DEVIATION DEPENDENT ON DISTANCE

Unlike the overland curves of Fig. 2, the oversea tropospheric wave propagation curves reproduced in Fig. 4 diverge with increasing distance. The standard deviation of the time variations implicit in this family of curves is therefore dependent on distance.

In a similar family of curves conforming to a strictly Gaussian distribution the separations between the curves possess the following magnitudes:

1% to 5%	$0{\cdot}6814\ \sigma$
1% to 10%	$1{\cdot}0448\ \sigma$
1% to 50%	$2{\cdot}3263\ \sigma$

where σ is the standard deviation. Unfortunately, the experimental curves of Fig. 4 do not conform to this distribution. They are, moreover, deficient in that they do not give the median value of field strength for distances beyond 400 km. They therefore need adapting before they can be used for multiple interference calculations.

The time probabilities of interest in providing protection against interference lie between 90% and 99%. Tropospheric field strengths are therefore required in the probability range 10% to 1%. There is in Fig. 4 an experimental curve at the 5% probability level, midway in the range of interest. If the separation between this and the 1% curve at any distance is treated as a datum, the standard deviation for that distance may be calculated (assuming a Gaussian distribution) by dividing the separation between the curves by 0·6814, and from values so obtained curves for other probability levels may be derived.

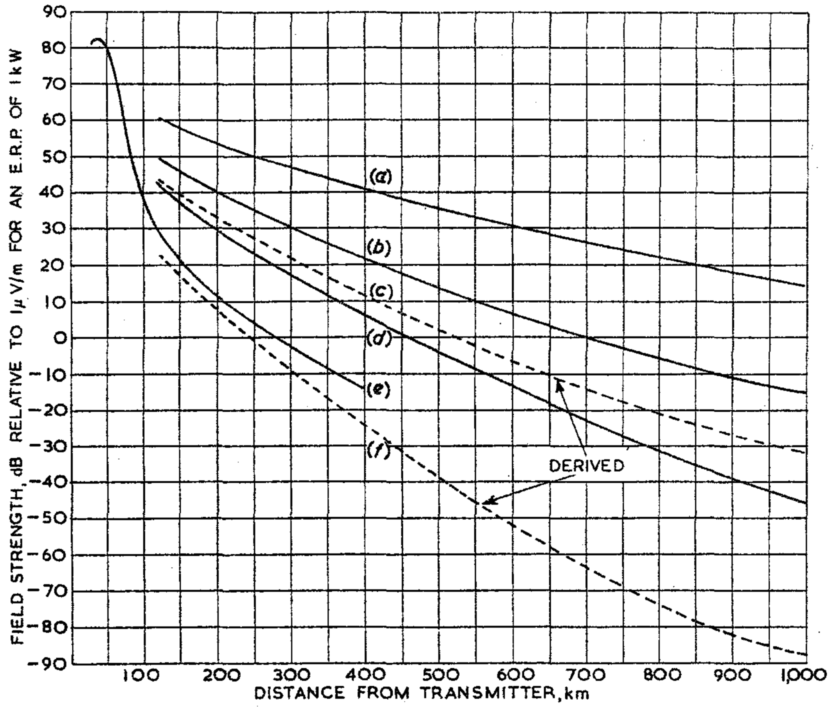

Fig. 4 - Field strength of the tropospheric wave in Bands IV and V for oversea paths

(*a*) for 1% of the time (experimental C.C.I.R. curve)
(*b*) for 5% of the time (experimental C.C.I.R. curve)
(*c*) for 10% of the time (computed for a Gaussian distribution)
(*d*) for 10% of the time (experimental C.C.I.R. curve)
(*e*) for 50% of the time (experimental C.C.I.R. curve)
(*f*) for 50% of the time (computed for a Gaussian distribution)

Curve (*f*) is the reference median

Two derived curves, at the 50% and 10% probability levels, are shown in Fig. 4. The 50% curve is the 'Reference Median'. Field strengths in the probability range 1% to 10%, calculated as for a Gaussian distribution from this reference, will be in close agreement with experimental values. The maximum error will occur at the 10% level, and to indicate its magnitude, the 10% derived curve is also shown in Fig. 4. Clearly the reference median must not be used to calculate protection probabilities lower than 90%.

The standard deviations derived from the separation between the 1% and 5% curves of Fig. 4 are plotted against distance in Fig. 5. Also plotted is the standard deviation for variations in time-location, a quantity similarly dependent on distance, since it incorporates the standard deviation of the time variations. For the curve drawn, the standard deviation of the variations with location was taken at a value 8 dB.

A table of logarithmic indices limited to a single value of standard deviation is of little use in multiple interference problems in which the standard deviation varies. A suitable form of tabulation for such calculations is one in which the argument is x/σ, where x is the deviation from the median. Table 3 is an example of this. Except for the added step of converting the deviation in decibels to a fraction of a standard deviation, the calculating procedure is identical with the example given.

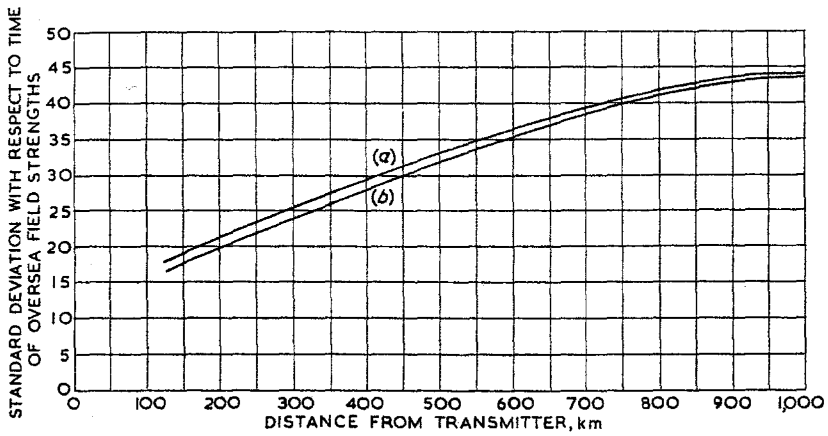

Fig. 5 - Standard deviation with respect to time and time-location as a function of distance, for oversea paths in Bands IV and V

(*a*) Time-location
(*b*) Time

13. CONCLUSION

Field strength variations occur with changes of location and in the passage of time, and the time-location frame of reference both reflects the physical reality of these variations and enables the probabilities associated with them to be comprehensively expressed. A method of estimating multiple co-channel interference in time-location has been described and illustrated by an example. The logarithmic index is recommended as a measure of the standard of protection.

The reliability of estimates of protection made by the method described depends on using the proper value of standard deviation for the relative variations between the wanted and unwanted signals. Short experiments have been described in which a representative value of the coefficient of correlation was found to be 0·5 for location as well as time variations.

An advantage of estimating protection in time-location lies in the flexibility with which the standard of protection expressed by the logarithmic index may be interpreted as a range of pairs of probabilities in time and with location.

14. ACKNOWLEDGEMENT

The work which led to the method of estimating multiple interference proposed in this paper was done jointly with Mr. G.F. Swann of the Post Office Engineering Department as a study programme for the Television Advisory Committee. The author wishes to thank Mr. Swann for his very cordial collaboration, and for the practical and statistical tests which he devised to check the accuracy of the proposed method.

15. REFERENCES

1. Report of the C.C.I.R. Committee of Experts to prepare for the European V.H.F./U.H.F. Broadcasting Conference (Document 64), Part 3.

2. Report of the C.C.I.R. Committee of Experts to prepare for the European V.H.F./U.H.F. Broadcasting Conference (Document 64), Part 1.

3. 'Report of the Ad Hoc Committee', Federal Communications Commission (Dockets Nos. 8736, 8975 and 9175), for the evaluation of the radio propagation factors concerning the television and frequency modulation services in the frequency range between 50 and 250 Mc/s. Vol. II, 7th July 1950, Appendix A, page 11.

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TABLE 1

Normal Distribution for a Standard Deviation of 11·3 dB

DEVIA-TION dB	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIA-TION dB	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIA-TION dB	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIA-TION dB	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX
0	50·0	30103	10·0	81·2	9049	20·0	96·2	1700	30·0	99·60	173
0·5	51·8	28597	10·5	82·4	8428	20·5	96·5	1500	30·5	99·65	151
1·0	53·5	27144	11·0	83·5	7839	21·0	96·8	1392	31·0	99·70	132
1·5	55·3	25744	11·5	84·6	7283	21·5	97·1	1258	31·5	99·73	115
2·0	57·0	24395	12·0	85·6	6758	22·0	97·4	1134	32·0	99·77	100
2·5	58·8	23096	12·5	86·6	6264	22·5	97·7	1021	32·5	99·80	87
3·0	60·5	21847	13·0	87·5	5798	23·0	97·9	918	33·0	99·83	72
3·5	62·2	20648	13·5	88·4	5360	23·5	98·1	824	33·5	99·85	66
4·0	63·8	19496	14·0	89·2	4948	24·0	98·3	738	34·0	99·87	57
4·5	65·5	18391	14·5	90·0	4558	24·5	98·5	660	34·5	99·89	49
5·0	67·1	17333	15·0	90·8	4200	25·0	98·65	589	35·0	99·90	42
5·5	68·7	16320	15·5	91·5	3861	25·5	98·80	525	35·5	99·92	36
6·0	70·2	15349	16·0	92·2	3546	26·0	98·93	467	36·0	99·93	30
6·5	71·7	14422	16·5	92·8	3250	26·5	99·05	415	36·5	99·94	26
7·0	73·2	13537	17·0	93·4	2977	27·0	99·16	368	37·0	99·95	23
7·5	74·7	12693	17·5	93·9	2721	27·5	99·25	326	37·5	99·96	19
8·0	76·1	11889	18·0	94·4	2485	28·0	99·34	288	38·0	99·96	17
8·5	77·4	11124	18·5	94·9	2264	28·5	99·42	254	38·5	99·97	14
9·0	78·7	10396	19·0	95·4	2061	29·0	99·49	223	39·0	99·97	11
9·5	80·0	9704	19·5	95·8	1873	29·5	99·55	197	39·5	99·98	9

TABLE 2

Interference at Brighton

Wanted Transmitter: Whitehawk Hill   Channel 2   Vertical Polarization   Offset +6·75 kc/s

1	2	3	4	5	6	7	8	9	10	11		12		13	
UNWANTED TRANSMITTER	POL.	OFFSET	BEARING FROM UNWANTED TRANSMITTER	AE. HT. ABOVE MEAN TERRAIN	DISTANCE	E.R.P. dB REL. 1 kW	50% FS dB(μV/m) for 1 kW	REQUIRED P.R.	FIELD STRENGTH PROTECTED AGAINST MEDIAN INTERFERENCE	FIELD STRENGTH TO BE PROTECTED					
										64 dB		65 dB		66 dB	
		kc/s		m	km			dB	dB	Diff.	Index	Diff.	Index	Diff.	Index
Holme Moss	V	0	160°	456	330	+20	-7	35	48	-16	3546	-17	2977	-18	2485
North Hessary Tor	V	-6·75	83°	305	282	+15	-3	35	47	-17	2977	-18	2485	-19	2061
Rosemarkie	H	-6·75	160°	183	810	-2	-49	25	-26	-90	-	-91	-	-92	-
Londonderry	H	+6·75	131°	168	676	-3	-38	35	-6	-70	-	-71	-	-72	-
Dover	V	-6·75	251°	91	113	-6	+17	35	46	-18	2485	-19	2061	-20	1700
Oxford	H	+47·25	146°	152	125	-10	+17	35	42	-22	1134	-23	918	-24	738
Ballachulish	V	+16·875	150°	0	740	-3	< -49	45	< -7	< -71	-	< -72	-	< -73	-
Berwick	H	+37·12	165°	91	580	0	-31	35	4	-60	-	-61	-	-62	-

		64 dB	65 dB	66 dB
Index Totals		10142	8441	6984
Time-Location Probability	%	79·1	82·4	85·1
Time Probability for 50% of Locations	%	87·5*	90·3	92·9

*Extrapolated

TABLE 3

Universal Table of Normal Distribution for a Standard Deviation (σ)

DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX
0·00	50·0	30103	0·30	61·8	20907	0·60	72·6	13921	0·90	81·6	8834
0·01	50·4	29757	0·31	62·2	20640	0·61	72·9	13723	0·91	81·9	8694
0·02	50·8	29415	0·32	62·6	20375	0·62	73·2	13528	0·92	82·1	8554
0·03	51·2	29075	0·33	62·9	20114	0·63	73·6	13333	0·93	82·4	8417
0·04	51·6	28739	0·34	63·3	19849	0·64	73·9	13140	0·94	82·7	8281
0·05	52·0	28403	0·35	63·7	19596	0·65	74·2	12951	0·95	82·9	8152
0·06	52·4	28163	0·36	64·1	19343	0·66	74·5	12763	0·96	83·1	8016
0·07	52·8	27745	0·37	64·4	19090	0·67	74·9	12577	0·97	83·4	7885
0·08	53·2	27420	0·38	64·8	18833	0·68	75·2	12353	0·98	83·6	7755
0·09	53·6	27095	0·39	65·2	18593	0·69	75·5	12211	0·99	83·9	7628
0·10	54·0	26773	0·40	65·5	18348	0·70	75·8	12029	1·00	84·1	7503
0·11	54·4	26456	0·41	65·9	18104	0·71	76·1	11848	1·01	84·4	7378
0·12	54·8	26141	0·42	66·3	17865	0·72	76·4	11677	1·02	84·6	7256
0·13	55·2	25828	0·43	66·6	17627	0·73	76·7	11503	1·03	84·9	7134
0·14	55·6	25519	0·44	67·0	17391	0·74	77·0	11331	1·04	85·1	7015
0·15	56·0	25210	0·45	67·4	17157	0·75	77·3	11162	1·05	85·3	6898
0·16	56·4	24907	0·46	67·7	16925	0·76	77·6	10994	1·06	85·5	6780
0·17	56·7	24605	0·47	68·1	16696	0·77	77·9	10826	1·07	85·8	6667
0·18	57·1	24304	0·48	68·4	16470	0·78	78·2	10662	1·08	86·0	6553
0·19	57·5	24007	0·49	68·8	16245	0·79	78·5	10500	1·09	86·2	6442
0·20	57·9	23713	0·50	69·1	16023	0·80	78·8	10343	1·10	86·4	6332
0·21	58·3	23421	0·51	69·5	15802	0·81	79·1	10180	1·11	86·7	6223
0·22	58·7	23125	0·52	69·8	15585	0·82	79·4	10024	1·12	86·9	6116
0·23	59·1	22911	0·53	70·2	15369	0·83	79·7	9869	1·13	87·1	6010
0·24	59·5	22560	0·54	70·5	15156	0·84	80·0	9715	1·14	87·3	5905
0·25	59·9	22278	0·55	70·9	14945	0·85	80·2	9564	1·15	87·5	5802
0·26	60·3	22000	0·56	71·2	14737	0·86	80·5	9414	1·16	87·7	5701
0·27	60·6	21722	0·57	71·6	14529	0·87	80·8	9268	1·17	87·9	5601
0·28	61·0	21449	0·58	71·9	14324	0·88	81·1	9121	1·18	88·1	5502
0·29	61·4	21382	0·59	72·2	14131	0·89	81·3	8976	1·19	88·3	5405

DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX
1·20	88·5	5309	1·50	93·3	3003	1·80	96·4	1589	2·10	98·2	783
1·21	88·7	5215	1·51	93·4	2943	1·81	96·5	1554	2·11	98·3	764
1·22	88·9	5121	1·52	93·6	2885	1·82	96·6	1520	2·12	98·3	745
1·23	89·1	5029	1·53	93·7	2826	1·83	96·6	1485	2·13	98·3	727
1·24	89·3	4939	1·54	93·8	2770	1·84	96·7	1452	2·14	98·4	709
1·25	89·4	4849	1·55	93·9	2713	1·85	96·8	1419	2·15	98·4	686
1·26	89·6	4761	1·56	94·1	2658	1·86	96·9	1388	2·16	98·5	669
1·27	89·8	4674	1·57	94·2	2604	1·87	96·9	1357	2·17	98·5	656
1·28	90·0	4588	1·58	94·3	2551	1·88	97·0	1326	2·18	98·5	640
1·29	90·1	4506	1·59	94·4	2500	1·89	97·1	1295	2·19	98·6	623
1·30	90·3	4417	1·60	94·5	2448	1·90	97·1	1266	2·20	98·6	608
1·31	90·5	4340	1·61	94·6	2397	1·91	97·2	1237	2·21	98·6	593
1·32	90·7	4259	1·62	94·7	2348	1·92	97·3	1208	2·22	98·7	577
1·33	90·8	4180	1·63	94·8	2300	1·93	97·3	1176	2·23	98·7	563
1·34	91·0	4102	1·64	95·0	2250	1·94	97·4	1152	2·24	98·7	549
1·35	91·1	4024	1·65	95·1	2204	1·95	97·4	1126	2·25	98·8	534
1·36	91·3	3948	1·66	95·2	2157	1·96	97·5	1100	2·26	98·8	521
1·37	91·5	3874	1·67	95·3	2111	1·97	97·6	1074	2·27	98·8	507
1·38	91·6	3800	1·68	95·4	2067	1·98	97·6	1049	2·28	98·9	493
1·39	91·8	3728	1·69	95·4	2022	1·99	97·7	1024	2·29	98·9	480
1·40	91·9	3657	1·70	95·5	1980	2·00	97·7	998	2·30	98·9	468
1·41	92·1	3586	1·71	95·6	1937	2·01	97·8	976	2·31	99·0	456
1·42	92·2	3518	1·72	95·7	1896	2·02	97·8	953	2·32	99·0	444
1·43	92·4	3450	1·73	95·8	1855	2·03	97·9	930	2·33	99·0	432
1·44	92·5	3383	1·74	95·9	1814	2·04	97·9	908	2·34	99·0	421
1·45	92·6	3317	1·75	96·0	1755	2·05	98·0	885	2·35	99·1	410
1·46	92·8	3252	1·76	96·1	1737	2·06	98·0	864	2·36	99·1	399
1·47	92·9	3188	1·77	96·2	1699	2·07	98·1	843	2·37	99·1	389
1·48	93·1	3127	1·78	96·2	1662	2·08	98·1	823	2·38	99·1	378
1·49	93·2	3064	1·79	96·3	1625	2·09	98·2	803	2·39	99·2	369

DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION x/σ	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX	DEVIATION	PERCENTAGE PROBABILITY	LOGARITHMIC INDEX
2·40	99·2	358	2·70	99·7	150	3·0	99·87	59			
2·41	99·2	348	2·71	99·7	146						
2·42	99·2	338	2·72	99·7	141	3·1	99·90	42			
2·43	99·2	330	2·73	99·7	136						
2·44	99·3	321	2·74	99·7	132	3·2	99·93	30			
2·45	99·3	312	2·75	99·7	128	3·3	99·95	20			
2·46	99·3	303	2·76	99·7	125						
2·47	99·3	294	2·77	99·7	122	3·4	99·97	14			
2·48	99·3	286	2·78	99·7	118						
2·49	99·4	278	2·79	99·7	115	3·5	99·98	9			
2·50	99·4	270	2·80	99·7	111						
2·51	99·4	263	2·81	99·8	108						
2·52	99·4	255	2·82	99·8	104						
2·53	99·4	248	2·83	99·8	101						
2·54	99·5	241	2·84	99·8	97						
2·55	99·5	235	2·85	99·8	94						
2·56	99·5	227	2·86	99·8	91						
2·57	99·5	222	2·87	99·8	89						
2·58	99·5	216	2·88	99·8	87						
2·59	99·5	209	2·89	99·8	84						
2·60	99·5	203	2·90	99·8	81						
2·61	99·5	198	2·91	99·8	78						
2·62	99·6	192	2·92	99·8	76						
2·63	99·6	186	2·93	99·8	74						
2·64	99·6	181	2·94	99·8	72						
2·65	99·6	175	2·95	99·8	70						
2·66	99·6	170	2·96	99·8	67						
2·67	99·6	165	2·97	99·9	65						
2·68	99·6	160	2·98	99·9	63						
2·69	99·6	155	2·99	99·9	61						

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